The Boon of Gene-*Culture* Interaction for Effective Evolutionary Multitasking

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Abstract. Multifactorial optimization (MFO) is a recently proposed paradigm for *evolutionary multitasking* that is inspired by the possibility of harnessing underlying synergies between outwardly unrelated optimization problems through the process of *implicit genetic transfer*. In contrast to traditional single-objective and multi-objective optimization, which consider only a single problem in one optimization run, MFO aims at solving multiple optimization problems simultaneously. Through comprehensive empirical study, MFO has demonstrated notable performance on a variety of complex optimization problems. In this paper, we take a step towards better understanding the means by which MFO leads to the observed performance improvement. In particular, since (a) genetic and (b) cultural transmission across generations form the crux of the proposed *evolutionary multitasking engine*, we focus on how their interaction (i.e., gene-culture interaction) affects the overall efficacy of this novel paradigm.

1 Introduction

Evolutionary algorithms (EAs) are generic population-based metaheuristics for optimization that employ mechanisms inspired by biological evolution, namely, Darwinian principles of *Natural Selection* or *Survival of the Fittest* [1]. Through computational analogues of sexual reproduction and mutation, EAs are capable of exploring and exploiting promising regions of the search space, with the survival pressure encouraging evolution of the entire population towards fitter regions of the objective function landscape [2]. In the literature, EAs have demonstrated powerful search capability and have been successfully applied on a wide variety of real-world problems [3].

Over the past few decades, EAs have attracted much research attention, with several variants proposed for single-objective optimization [4], multi-objective optimization [5], and many-objective optimization [6]. It is worth noting that the majority of these works focus on efficiently dealing with only a single problem at a time. Seldom has an attempt been made to multitask, i.e., to solve

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multiple optimization problems (or multiple *tasks*) simultaneously using a single population of evolving individuals. It is only very recently that A. Gupta *et al.* have proposed a new paradigm, labeled *multifactorial optimization* (MFO) [7], that attempts to harness the intrinsic potential for *evolutionary multitasking* possessed by population-based search strategies (here the term 'multifactorial' is used to imply that every task contributes a different *factor* influencing the evolution of the population). For example, consider a scenario where two popular benchmarks from continuous optimization, such as the Rastrigin function and the Ackley function, are to be solved simultaneously. In such cases, evolutionary multitasking provides the scope for *autonomously* exploiting underlying synergies (or what we term as the *latent genetic complementarities* [7]) between otherwise independent tasks, through the process of *implicit genetic transfer*.

To realize the MFO paradigm, a novel algorithm, namely, the *multifactorial* evolutionary algorithm (MFEA), has also been proposed in [7]. The MFEA is inspired by bio-cultural models of *multifactorial inheritance* [8], which contend that the complex developmental traits among offspring are influenced by geneculture interactions. The computational equivalent of multifactorial inheritance, for the purpose of efficient evolutionary multitasking, is established by considering each optimization task to create a *distinct environment* in which offspring can be reared. In other words, from the standpoint of the MFEA, multitasking leads to the coexistence of multiple blocks of cultural bias (or memes [9]), one corresponding to each task. The subsequent evolution of encoded individuals in the composite landscape is simulated through an interplay of genetic and cultural transmission, where cultural aspects are manifested by two major components of the MFEA acting in concert, namely, (a) non-random or assortative mating: which states that individuals prefer to mate with those sharing a similar cultural background, and (b) vertical cultural transmission: which states that the phenotype of an offspring is strongly influenced by that of its parents. While the basic structure of the proposed algorithm is similar to a classical EA, it is augmented by the aforementioned features that are borrowed from the models of multifactorial inheritance. Using the proposed algorithm, the MFO paradigm has been thoroughly studied in [7] via several computational experiments. These included multitasking across continuous optimization tasks, or combinatorial optimization tasks, or even a mixture of combinatorial and continuous tasks (cross-domain multitasking). In the majority of cases, MFEA demonstrated noteworthy performance by accelerating convergence for complex optimization tasks.

Bearing in mind the need for future algorithmic developments in the field of MFO, we find it essential, at this juncture, to investigate and fully acknowledge the key contribution of gene-culture interaction while designing effective *evolutionary multitasking engines*. To this end, in this paper, we present a variant of the MFEA, labeled as *polygenic evolutionary algorithm* (PGEA), which curtails the cultural aspects of the evolutionary process as are manifested in the models of multifactorial inheritance. On comparing the performance of the MFEA and the PGEA on the same set of benchmark instances, it becomes possible

to decipher the benefits to the multitasking procedure provided by gene-culture interaction.

The remainder of the paper is organized as follows. Section II covers the preliminaries. It introduces the basic concepts of MFO, describes the MFEA, and presents an overview of cultural transmission in multifactorial inheritance. In Section III, we describe the PGEA and discuss our strategy for investigating and verifying the efficacy of the cultural aspects of the MFEA. In Section IV, computational experiments are carried out on a variety of benchmark functions from continuous optimization. These serve the purpose of highlighting the key contribution of gene-culture interaction towards effective evolutionary multitasking. Finally, Section V concludes the paper by summarizing the presented work.

2 Preliminaries

In this Section, we present an overview of the basic concepts in evolutionary multitasking, as have been proposed in [7].

2.1 Multifactorial Optimization (MFO)

Consider a scenario where K distinct optimization tasks are presented simultaneously to a single evolutionary solver. Let the j^{th} task be denoted as T_j , and the dimensionality of its search space \mathbf{X}_j be D_j . Without loss of generality, all tasks are assumed to be minimization problems, with the objective function of task T_j being given by $f_j : \mathbf{X}_j \to \mathbb{R}$. In such a setting, MFO is defined as an evolutionary multitasking paradigm that builds on the implicit parallelism [22] of population-based search with the aim of concurrently finding $\{\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_K\} = \arg\min\{f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots, f_K(\mathbf{x})\}$. Here, \mathbf{x}_j denotes a feasible solution in \mathbf{X}_j . Note that each f_j is treated as an additional factor influencing the whole evolutionary process. For this reason, the composite problem is referred to as a K-factorial problem.

The fundamentals of designing an EA are based on the Darwinian principle of natural selection. Hence, in order to develop a suitable algorithm for MFO, it is necessary to first conceive a valid measurement to evaluate the fitness of individuals in a multitasking environment. To this end, the following set of properties are defined for every individual p_i , where $i \in 1, 2, ..., |P|$, in a population P:

- Factorial Rank: The factorial rank r_{ij} of p_i on task T_j is simply the index of p_i in the list of population members sorted in ascending order with respect to f_j .
- Scalar Fitness: The list of factorial ranks $\{r_{i1}, r_{i2}, \cdots, r_{iK}\}$ of an individual p_i is reduced to a scalar fitness φ_i based on its best rank over all tasks; *i.e.* $\varphi_i = 1/\min\{r_{i1}, r_{i2}, \cdots, r_{iK}\}.$

- Skill Factor: The skill factor τ_i of p_i is the one task, amongst all other tasks in MFO, on which the individual is the most effective, *i.e.* $\tau_i = \arg \min_j \{r_{ij}\},$ where $j \in \{1, 2, \dots, K\}.$

Based on the definition of scalar fitness, the comparison between individuals can be achieved in a straightforward manner. For instance, an individual p_a is considered to dominate individual p_b in multifactorial sense simply if $\varphi_a > \varphi_b$.

Also, it is clear that the aforementioned fitness assignment and comparison procedure guarantees that if an individual p^* maps to the global optimal of any task, then, $\varphi_* \geq \varphi_i$ for all $i \in \{1, 2, \dots, |P|\}$. Therefore, the evolutionary environment built under the above definitions is indeed compatible with the ensuing definition of *multifactorial optimality*.

- Multifactorial Optimality: An individual p^* is considered optimum in multifactorial sense iff $\exists j \in \{1, 2, \dots, K\}$ such that $f_j^* \leq f_j(\boldsymbol{x}_j)$, for all feasible $\boldsymbol{x}_j \in \boldsymbol{X}_j$.

2.2 An Overview of the Multifactorial Evolutionary Algorithm

The MFEA is inspired by the bio-cultural models of multifactorial inheritance. The algorithm is in fact classified under the broad umbrella of *memetic computation* [9, 10] as it considers the transmission of biological as well as cultural building blocks (genes and memes) [11, 12] from parents to their offspring. In particular, cultural effects are incorporated via two aspects of multifactorial inheritance acting in concert, namely (a) assortative mating and (b) vertical cultural transmission.

The basic structure of the MFEA is presented in Algorithm 1. Details of its various distinctive features are discussed next.

Algorithm 1 Multifactorial evolutionary algorithm.

- 1: Generate an initial population of solutions and store it in *current-pop*.
- 2: Evaluate every solution with respect to every optimization task in the multitasking environment.
- 3: Compute the skill factor of each individual.
- 4: while (stopping conditions are not satisfied) do
- 5: Apply genetic operators on *current-pop* to generate an *offspring-pop* (see Algorithm 2).
- 6: Evaluate the individuals in *offspring-pop* for certain optimization tasks only (see Algorithm 3).
- 7: Concatenate offspring-pop and current-pop to form an intermediate-pop.
- 8: Re-compute the scalar fitness and skill factor of all individuals.
- 9: Select the fittest individuals from *intermediate-pop* to survive into the next generation and form the new *current-pop*.

10: end while

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2.3 Chromosome description and decoding procedure

Assuming there to be K optimization tasks, we define a unified search space Y with dimensionality $(D_{multitask})$ equal to $\max_j \{D_j\}$. Thus, during population initialization, every individual is assigned a vector of $D_{multitask}$ random-keys [13, 14] which lie in the fixed range [0, 1]. This vector constitutes the chromosome of that individual. While addressing task T_j , only the first D_j random-keys of the chromosome are considered.

There is a strong theoretical motivation behind using the aforementioned encoding scheme. In particular, it is considered to be an effective means of accessing the power of population-based search. As the *schemata* (or genetic building blocks) [15] corresponding to different optimization tasks are contained within a unified pool of genetic material, they get processed by the EA in parallel. Most importantly, this encourages the discovery and implicit transfer of useful genetic material from one task to another in an efficient manner. Moreover, as a single individual in the population may inherit genetic building blocks corresponding to multiple optimization task, the analogy with multifactorial inheritance becomes more meaningful.

Given a chromosome $\boldsymbol{y} \in \boldsymbol{Y}$, a decoding scheme must first be employed to transform \boldsymbol{y} into a meaningful task-specific solution representation. In the case of continuous optimization, this can be achieved by linearly mapping each randomkey from the unified space to the original search space of the optimization task. For instance, consider a task T_j in which the i^{th} variable (x_i) is bounded in the range $[L_i, U_i]$. If the i^{th} random-key of a chromosome \boldsymbol{y} takes value $y_i \in [0, 1]$, then the decoding procedure is given by $x_i = L_i + (U_i - L_i) \cdot y_i$

2.4 Cultural aspects of the MFEA

In the MFEA, we interpret the skill factor (τ) of an individual as a computational representation of its cultural background. Accordingly, while simulating genetic operations (via crossover and mutation), the phenomenon of assortative mating (which states that individuals prefer to mate with those sharing a similar cultural background) is enforced by prescribing a set of conditions that must be satisfied for two randomly selected parent candidates to undergo crossover. A summary is provided in Algorithm 2. The occurrence of assortative mating in the natural world is used in the models of multifactorial inheritance to explain pedigreed traits that extend over several generations [8]. In the case of the MFEA, we introduce a tunable parameter called the random mating probability (rmp) which follows the principle of assortative mating and is used to balance exploration and exploitation during evolution of individuals in the search space. Setting $rmp \approx 0$ implies that only culturally alike individuals are allowed to crossover, while setting $rmp \approx 1$ permits completely random cross-cultural mating. In the former case, the predominantly intra-cultural mating and the small genetic variations produced by mutation (see Algorithm 2) facilitate the scanning of confined regions of the search space. As a result however, there is always the tendency for solutions to get trapped in local optima. On the other hand,

Algorithm 2 Assortative mating

1: for i = 1 : |P|/2 do 2: Randomly select two parents P_1 and P_2 from *current-pop*. 3: Generate a random number rand between 0 and 1. 4: if $(\tau_1 == \tau_2)$ or (rand < rmp) then 5:Parents P_1 and P_2 crossover to give two offspring individuals C_1 and C_2 . 6: else 7: P_1 is mutated slightly to give an offspring C_1 . 8: P_2 is mutated slightly to give an offspring C_2 . 9: end if Append C_1 and C_2 to offspring-pop. 10:11: end for

when rmp is sufficiently greater than 0, the increased cross-cultural mating leads to the creation of offspring with diverse genetic properties, thereby facilitating the escape from local optima. In addition, it is contended that exclusive mating between individuals belonging to the same cultural background could lead to the loss of good and diverse genetic material available from other cultural backgrounds. In Algorithm 1, notice that the MFEA is bootstrapped by evalu-

Algorithm 3 Vertical cultural transmission

- 1: Consider an offspring C which either has 2 parents P_1 and P_2 , or a single parent P_1 (or P_2) (see Algorithm 2).
- 2: if (C has 2 parents) then
- 3: Generate a random number *rand* between 0 and 1.
- 4: **if** (rand < 0.5) **then**
- 5: C imitates $P_1 \to \text{Evaluate}$ and locally improve C with respect to task τ_1 (skill factor P_1).
- 6: else
- 7: C imitates $P_2 \to \text{Evaluate}$ and locally improve C with respect to task τ_2 (skill factor P_2).
- 8: end if
- 9: else
- 10: C is evaluated and locally improved with respect to task τ_1 (or τ_2).
- 11: end if
- 12: Objective function values of C with respect to all unevaluated tasks are artificially set to ∞ .

ating every individual in the initial population with respect to every task in the multitasking environment. However, it is evident that carrying out exhaustive evaluations in all subsequent generations is likely to be computationally too expensive. For that reason, it is considered practical for an offspring to only be evaluated for a particular task on which it is most likely to be effective. The algorithmic realization of the aforementioned notion is achieved via a *selective imitation* strategy [7] as a form of vertical cultural transmission (see Algorithm

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3). Accordingly, an offspring in the MFEA is only evaluated with respect to the task at which at least one of its parents is highly skilled. In other words, the offspring *randomly imitates* the skill factor (or cultural background) of any one of its parents. Furthermore, *every offspring undergoes local improvements* with respect to the skill factor that it chooses to imitate (details of the local search algorithm shall be provided in Section 4). Notice that since the genetic composition of an offspring is a combination of the genetic material of its parents, it is reasonable to expect its skill factor to liken that of its parents.

A crucial outcome emerges from the combined effect of assortative mating and vertical cultural transmission. On occasions when parents with different skill factors happen to crossover, a multicultural environment is created for offspring to be reared in. In such situations, it is possible for an offspring that is genetically closer to one parent to imitate or be culturally influenced by the other. It is this feature of the MFEA that leads to implicit genetic transfer across tasks. Refined genetic material created within individuals of a particular skill factor, if also useful for a different task, can be effectively transferred via the proposed mechanism.

3 The Polygenic Evolutionary Algorithm

As described above, assortative mating and vertical cultural transmission represent the core forms of cultural interaction in the MFEA. In order to understand their importance towards effective evolutionary multitasking, we herein propose an alternate *polygenic evolutionary algorithm* (PGEA) which curtails the cultural aspects of the evolutionary process as prevalent in the MFEA. Thus, in comparison to the PGEA, the credit of any improvement in performance achieved by the MFEA can be entirely assigned to gene-culture interactions.

In the PGEA, the first step to removing cultural bias is taken by ignoring the phenomenon of assortative mating. Thus, any two members of the PGEA population, regardless of whether they possess the same skill factor or not, are allowed to mate freely. In other words, the value of *rmp* is essentially fixed at 1, implying that uninhibited cross-cultural mating is allowed to occur; which is accompanied by probabilistic mutation of the generated offspring. Secondly, the effect of vertical cultural transmission is minimized by permitting the generated offspring to randomly select any task for evaluation regardless of the skill factor(s) of their parents (i.e., the strategy of selective imitation is also ignored). For a complete overview of the incorporated modifications, see Algorithm 4. The ramifications of these modifications towards the overall performance of the multitasking engine shall be investigated in the next section.

4 Empirical Study

The main aim behind the computational experiments is to better understand the effects of gene-culture interactions towards the overall efficacy of evolutionary multitasking. The simplest way of acquiring a qualitative understanding of the

Algorithm 4 Genetic mechanisms of the polygenic evolutionary algorithm

- 1: Consider two parents P_1 and P_2 randomly selected from *current-pop*.
- 2: P_1 and P_2 crossover to give two offspring solutions C_1 and C_2 .
- 3: Offspring C_1 and C_2 may be slightly mutated with a predefined probability.
- 4: Each offspring is evaluated for any one randomly selected task (individual learning will be applied here). The objective values of the offspring with respect to all unevaluated tasks are artificially set to ∞ .

effects is to compare the performance of the MFEA (which incorporates a variety of cultural biases through assortative mating and vertical cultural transmission) and the PGEA (which removes all cultural bias). The aforementioned approach is therefore adopted herein.

4.1 Experimental setup and algorithmic specifications

In this section, we carry out several computational experiments on popular benchmark problems in continuous optimization. We consider a unimodal function: a) sphere (search region [100, 100]), and four complex multimodal functions [16]: b) shifted and rotated Rastrigin (search region [-5, 5]) (search region denotes the box constraint on every dimension), c) shifted and rotated Ackley (search region [-32, 32]), d) shifted and rotated Griewank (search region [-600, 600]) and e) rotated Weierstrass (search region [-0.5, 0.5]). The dimensionality of each of the benchmark functions is fixed at 30, and the rotation matrix corresponding to each function is randomly generated. During computational experiments, we combine any two of the benchmark functions together to form a single 2-factorial problem. Moreover, the instances are setup such that the global optima of the two constitutive tasks in a single MFO problem are largely separated (demonstrated in Fig. 1a). This ensures that there is no *apparent* source of synergy (or genetic complementarity) between the tasks.

In Table 1, we list the descriptions of four pairs of 2-factorial problems that have been considered in this paper. The second column of the table states the combined functions. For example, $(Task_1, Task_2) \equiv (Sphere, Rastrigin)$ implies that the sphere function and the Rastrigin function have been combined into a single MFO run. The third column of Table 1 represents the location of the global optimum of $Task_1$, while the fourth column represents the location of the global optimum of $Task_2$. With regard to the MFEA and the PGEA, we set the population size to 100 individuals which are evolved over 500 generations. The rmp (which only occurs in the MFEA) is configured to 0.3 in all experiments so as to allow sufficient cross-cultural mating. With regard to the variation operators [17], we employ the Simulated Binary Crossover (SBX) [18] and Gaussian mutation operators throughout. Particularly, in the PGEA, the probability of mutation was kept fixed at 10%. Further, in order to facilitate the discovery of high quality solutions, we include a BFGS quasi-Newton individual learning step into each task evaluation call (note that learning proceeds in the spirit of Lamarckism [3, 19]). We realize that hybridizing EAs with individual learning

Table 1. Description of MFO test instances. $(Task_1, Task_2)$ implies that $Task_1$ and $Task_2$ are combined in a single MFO run.

No.	MFO problems	Global optimum of $Task_1$	Global optimum of $Task_2$
1	(Sphere, Rastrigin)	$x_i = 50, \forall i$	$x_i = 0, \forall i$
2	(Ackley, Weierstrass)	$x_i = 20, \forall i$	$x_i = 0, \forall i$
3	(Griewank, Weierstrass)	$x_i = 300, \forall i$	$x_i = 0, \forall i$
4	(Sphere, Ackley)	$x_i = 50, \forall i$	$x_i = 0, \forall i$

(via local search) is traditionally perceived as a form of cultural evolution or as a *first generation memetic algorithm* [19, 20]. However, judging from the standpoint of multifactorial inheritance, in the present work, local search is not viewed as a separate source of cultural influence that acts over and above assortative mating and vertical cultural transmission.

4.2 Discussions

Figure 1a depicts a 1-D illustration of the separated sphere and Rastrigin functions. The convergence trends of the multimodal Rastrigin function in this scenario (given a 30-D search space), as obtained by the MFEA and the PGEA, is provided in Figure 1b. In addition, the figure also contains a third curve, labeled as SOEA, which represents a traditional single-objective optimization-based approach to solving the Rastrigin function (note that the SOEA employs identical variation operators and local search process as the MFEA and the PGEA). For fairness of comparison, the SOEA is also enhanced with the same Lamarckian local search algorithm as the MFEA and the PGEA. It is clear from the convergence trends that the performance of the MFEA and the PGEA far exceed that of the SOEA on this instance. The observation underpins our broader claim that provisions for enhanced population diversity and implicit genetic transfer, as facilitated by the evolutionary multitasking paradigm, are potentially invaluable tools for accelerating the convergence process of complex optimization tasks.

On further inspecting Figure 1b, it is observed that the performance achieved by the MFEA is even superior to that of the PGEA. This result provides strong evidence of the fact that gene-cultural interactions play an important role in improving convergence characteristics. As has been discussed in Section 2.4, the cultural aspects of the MFEA (manifested by assortative mating and vertical cultural transmission, acting in concert) lead to a favorable balance between exploration (via population diversification and genetic transfer during *controlled* cross-cultural mating) and exploitation (via assortative mating) of the search space. In contrast, in the PGEA, the removal of cultural bias disrupts the aforementioned balance. The uninhibited cross-cultural mating leads to excessive mixing of genes, eventually causing the loss of pedigreed high quality genetic material [7]. Moreover, by ignoring vertical cultural transmission, the systematic search of fitter regions of the objective function landscape is impeded. Therefore, it comes as little surprise that the performance of the resultant multitasking engine (i.e., the PGEA) is inferior to that of the MFEA. The convergence trends depicted in



Fig. 1. (a) 1-D illustration of separated Rastrigin and sphere functions in the unified search space, and (b) convergence trends of Rastrigin function in (*Sphere, Rastrigin*).



Fig. 2. The convergence trends of (a) Ackley function and (b) Weierstrass function in (*Ackley, Weierstrass*).

Figures 2-4 (corresponding to problem numbers 2-4 in Table 1) have similar qualitative characteristics as those presented in Figure 1. This empirical observation goes a long way towards further reinforcing our inferences as drawn previously. In Figure 2, the convergence trends of Ackley function (in Figure 2a) and the Weierstrass function (in Figure 2b) are presented, when solved in conjunction as (Ackley, Weierstrass). Note that both functions in this 2-factorial problem are complex and multimodal. Nevertheless, the convergence rate achieved by the MFEA is found to be accelerated in both cases, in comparison to the PGEA as well as the SOEA. Thus, it is contended that the provision for implicit genetic transfer, appropriately supervised by gene-culture interactions as prescribed by the models of multifactorial inheritance, allows the population to successfully exploit the landscape of multiple complex functions simultaneously, thereby efficiently bypassing obstacles to converge faster.

An important observation we also make here is that the performance of the PGEA proves to be even inferior to the SOEA in some of the examples (see Figure 2a and Figure 4). This shows that eliminating cultural bias altogether from an evolutionary multitasking engine can often be highly detrimental to its

overall performance. In other words, the benefits of gene-culture interaction are strongly highlighted in these examples.



Fig. 3. The convergence trends of Weierstrass function in (Weierstrass, Griewank).



Fig. 4. The convergence trends of Ackley function in (Sphere, Ackley).

5 Conclusion

The main aim of this paper was to show the importance of gene-culture interaction, as manifested in the bio-cultural models of multifactorial inheritance, towards the design of an effective multitasking engine. To this end, we have presented a pair of algorithms, namely, (a) the original multifactorial evolutionary algorithm (MFEA) which includes cultural biases, and (b) a new polygenic evolutionary algorithm (PGEA) which curtails all cultural aspects of the evolutionary process. The consistently superior performance of the MFEA, as compared to the PGEA (and also a traditional single-objective optimization approach), on a variety of benchmark problems in continuous optimization, has demonstrated that the incorporation of gene-culture interaction is indeed a pivotal aspect of effective evolutionary multitasking.

References

- 1. J. R. Koza. Genetic programming: on the programming of computers by means of natural selection, volume 1. MIT press, 1992.
- A. Konak, D. W. Coit, and A. E. Smith. Multi-objective optimization using genetic algorithms: A tutorial. *Reliability Engineering & System Safety*, 91(9):992–1007, 2006.
- Y. S. Ong and A. J. Keane. Meta-lamarckian learning in memetic algorithms. Evolutionary Computation, IEEE Transactions on, 8(2):99–110, 2004.
- T. Bäck, U. Hammel, and H. P. Schwefel. Evolutionary computation: Comments on the history and current state. *Evolutionary computation*, *IEEE Transactions* on, 1(1):3–17, 1997.
- C. M. Fonseca and P. J. Fleming. An overview of evolutionary algorithms in multiobjective optimization. *Evolutionary computation*, 3(1):1–16, 1995.
- H. Ishibuchi, N. Tsukamoto, and Y. Nojima. Evolutionary many-objective optimization: A short review. In *IEEE congress on evolutionary computation*, pages 2419–2426. Citeseer, 2008.
- A. Gupta, Y. S. Ong, and L. Feng. Multifactorial evolution. Evolutionary Computation, IEEE Transactions on, PP(99):1-1, 2015.
- J. Rice, C. R. Cloninger, and T. Reich. Multifactorial inheritance with cultural transmission and assortative mating. i. description and basic properties of the unitary models. *American journal of human genetics*, 30(6):618, 1978.
- X. Chen, Y. S. Ong, M. H. Lim, and K. C. Tan. A multi-facet survey on memetic computation. *IEEE Transactions on Evolutionary Computation*, 15(5):591–607, 2011.
- Y. S. Ong, M. H. Lim, and X. Chen. Research frontier-memetic computation—past, present & future. *IEEE Computational Intelligence Magazine*, 5(2):24, 2010.
- M. Iqbal, W. N. Browne, and M. Zhang. Reusing building blocks of extracted knowledge to solve complex, large-scale boolean problems. *Evolutionary Compu*tation, *IEEE Transactions on*, 18(4):465–480, 2014.
- R. Mills, T. Jansen, and R. A. Watson. Transforming evolutionary search into higher-level evolutionary search by capturing problem structure. *Evolutionary Computation, IEEE Transactions on*, 18(5):628–642, Oct 2014.
- J. C. Bean. Genetic algorithms and random keys for sequencing and optimization. ORSA journal on computing, 6(2):154–160, 1994.
- J. F. Gonçalves and M. GC Resende. Biased random-key genetic algorithms for combinatorial optimization. *Journal of Heuristics*, 17(5):487–525, 2011.
- A. H. Wright, M. D. Vose, and J. E. Rowe. Implicit parallelism. In *Genetic and Evolutionary Computation—GECCO 2003*, pages 1505–1517. Springer, 2003.
- Y. S. Ong, Z. Zhou, and D. Lim. Curse and blessing of uncertainty in evolutionary algorithm using approximation. In *Evolutionary Computation*, 2006. CEC 2006. *IEEE Congress on*, pages 2928–2935. IEEE, 2006.
- 17. D. B. Fogel. What is evolutionary computation? Spec., IEEE, 37(2):26-28, 2000.
- K. Deb and R. B. Agrawal. Simulated binary crossover for continuous search space. Complex Systems, 9(3):1–15, 1994.
- 19. R. Meuth, M. H. Lim, Y. S. Ong, and D. C. Wunsch II. A proposition on memes and meta-memes in computing for higher-order learning.
- C. C. António. A memetic algorithm based on multiple learning procedures for global optimal design of composite structures. *Memetic Computing*, 6(2):113–131, 2014.

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